Simulation of Energy Loss Straggling

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April 7, 2008

1 Introduction

Due to the statistical nature of ionisation energy loss, large fluctuations can occur in the amount of energy deposited by a particle traversing an absorber element. Continuous processes such as multiple scattering and energy loss play a relevant role in the longitudinal and lateral development of electromagnetic and hadronic showers, and in the case of sampling calorimeters the measured resolution can be significantly affected by such fluctuations in their active layers. ...

2 Vavilov theory

Vavilov[2] derived a more accurate straggling distribution by introducing the kinematic limit on the maximum transferable energy in a single collision, rather than using $E_{\max} = \infty$. Now we can write[1]:

$$f(\epsilon, \delta s) = \frac{1}{\xi}\phi_v\left(\lambda_v, \kappa, \beta^2\right)$$

where

$$\begin{aligned}
\phi_v\left(\lambda_v, \kappa, \beta^2\right) &= \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\phi(s)\, e^{\lambda s} ds \qquad c \geq 0 \\
\phi(s) &= \exp\left[\kappa(1+\beta^2\gamma)\right] \ \exp\left[\psi(s)\right], \\
\psi(s) &= s\ln\kappa + (s+\beta^2\kappa)\left[\ln(s/\kappa) + E_1(s/\kappa)\right] - \kappa e^{-s/\kappa},
\end{aligned}$$

and

$$\begin{aligned}
E_1(z) &= \int_z^{\infty} t^{-1}e^{-t}dt \qquad \text{(the exponential integral)} \\
\lambda_v &= \kappa\left[\frac{\epsilon - \bar{\epsilon}}{\xi} - \gamma' - \beta^2\right]
\end{aligned}$$

The Vavilov parameters are simply related to the Landau parameter by $\lambda_L = \lambda_v/\kappa - \ln\kappa$. It can be shown that as $\kappa \rightarrow 0$, the distribution of the variable λ_L approaches that of Landau. For $\kappa \leq 0.01$ the two distributions are already practically identical. Contrary to what many textbooks report, the Vavilov distribution does not approximate the Landau distribution for small κ, but rather the distribution of λ_L defined above tends to the distribution of the true λ from the Landau density function. Thus the routine `GVAVIV` samples the variable λ_L rather than λ_v. For $\kappa \geq 10$ the Vavilov distribution tends to a Gaussian distribution (see next section).

References

[1] B.Schorr. Programs for the Landau and the Vavilov distributions and the corresponding random numbers. Comp. Phys. Comm., 7:216, 1974.

[2] P.V.Vavilov. Ionisation losses of high energy heavy particles. Soviet Physics JETP, 5:749, 1957.

Document preamble

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\documentclass[paper=a4,fontsize=10pt,smallheadings]{scrartcl}

\usepackage[T1]{fontenc}
\usepackage{emerald}
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